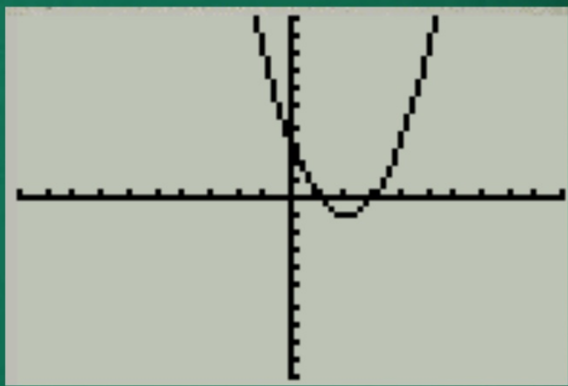
WARM UP

Graph $f(x) = x^2 - 4x + 3$

1. Domain:
2. Range:
3. Intercept
4. Intervals



OBJECTIVE

Students will be able to identify key features of their graphs, and graph rationals by hand using key features.

ESSENTIAL QUESTION

How do the graphs of rational functions compare to the graphs of polynomials?

THINK-PAIR-SHARE

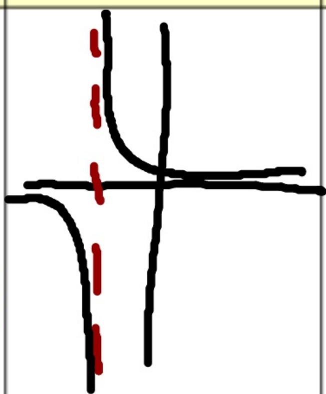
Submit on loose-leaf

1. Graph any polynomial function (e.g. $y=x^2-5$)
2. In the same window, graph any rational function (e.g. $y = \frac{x-2}{x^2-25}$).
3. What are some differences you notice between these two functions?
4. Finally, turn to your neighbor & share: what did you both find was different?

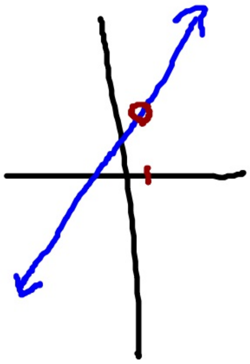
Why do rationals "break"?

$y = \frac{1}{x}$ $\quad x \neq 0$

Discontinuous: break in the Domain & Range

Function	Name	Graph	Characteristics
$y = \frac{x-2}{x^2-9}$	Infinite Discontinuity (asymptote)		The graph becomes greater and greater as it approaches a given x-value. $x \to n$ from right, $y \to \infty$; $x \to n$ from left, $y \to -\infty$

DISCONTINUITY…

Function	Name	Graph	Characteristics
$x^2 - 1$ $y = \frac{(x+1)(x-1)}{(x-1)}$	Point Discontinuity "Holes"		When there is a value in the domain for which the function is undefined, but the pieces of the graph match up. There is a hole in the graph.

GRAPHING RATIONALS

EX: $f(x) = \dfrac{x+4}{x^2+7x+12} = \dfrac{x+4}{(x+3)(x+4)}$

Step 1: Factor numerator and denominator

Identifying Restrictions & Intercepts

Graph: $f(x) = \dfrac{x + 4}{(x + 4)(x + 3)} \neq 0$

$x \neq -4$ $x = -3$

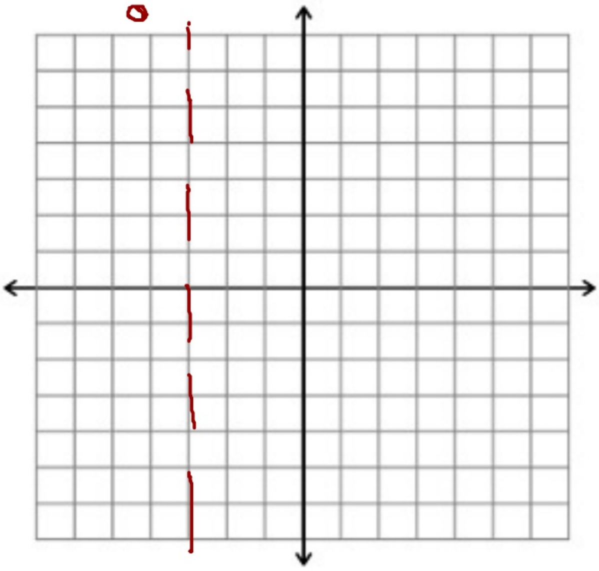
Step 2: Set the denominator ≠ 0 & solve to find vertical asymptotes and holes.

$x \neq -4$, hole $x \neq -3$, VA

Holes	**Root(s) are the same in numerator & denominator & would cancel**
Vertical Asymptotes	**Root(s) don't match any in the numerator**

Identifying Restrictions & Intercepts

Graph: $f(x) = \dfrac{x + 4}{x^2 + 7x + 12}$



Identifying Restrictions & Intercepts

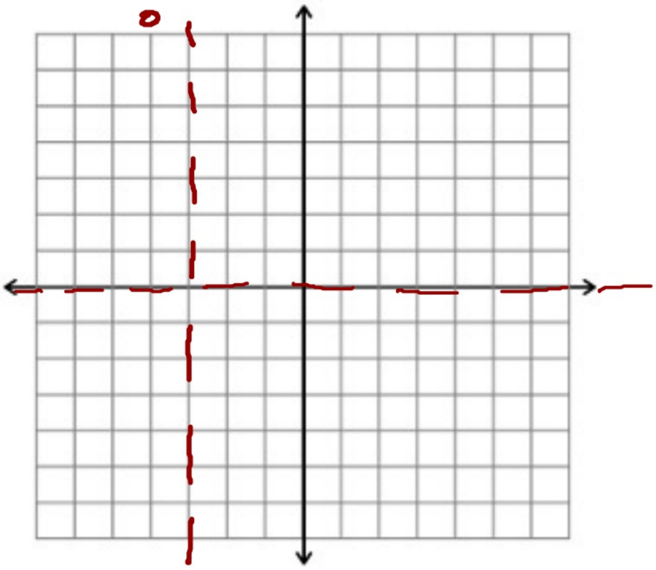
Graph: $f(x) = \dfrac{x^{1} + 4}{x^{2} + 7x + 12}$

Step 3: Use rules to determine horizontal asymptotes.

Horizontal Asymptotes	Degree of Numerator Higher than Denominator	None
	Degree of Denominator Higher than Numerator	$y = 0$
	Degree of Numerator and Denominator Equal	$y = 1$

Identifying Restrictions & Intercepts

Graph: $f(x) = \dfrac{x + 4}{x^2 + 7x + 12}$



Identifying Restrictions & Intercepts

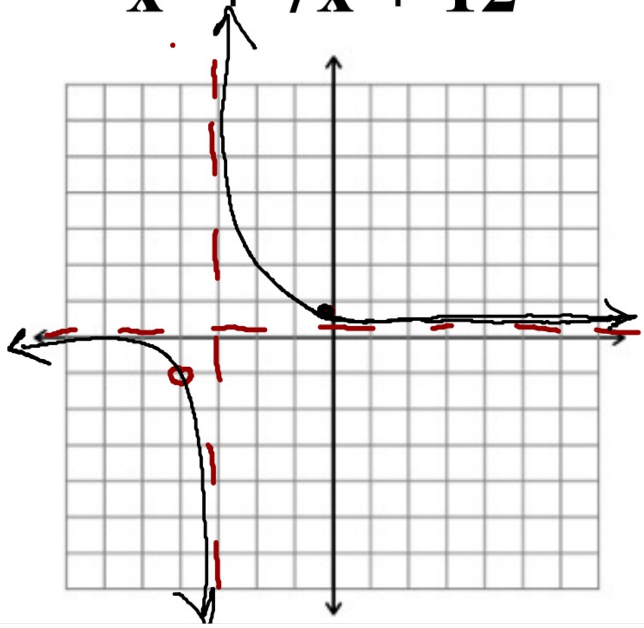
Graph: $f(x) = \dfrac{x + 4}{(x + 4)(x + 3)}$

Step 4: Set x=0 to find y-intercept(s).

$$f(x) = \frac{0 + 4}{(0 + 4)(0 + 3)} = \frac{4}{12} = \frac{1}{3}$$

Identifying Restrictions & Intercepts

Graph: $f(x) = \dfrac{x + 4}{x^2 + 7x + 12}$



Identifying Restrictions & Intercepts

Graph: $f(x) = \dfrac{x + 4}{x^2 + 7x + 12} = 0$

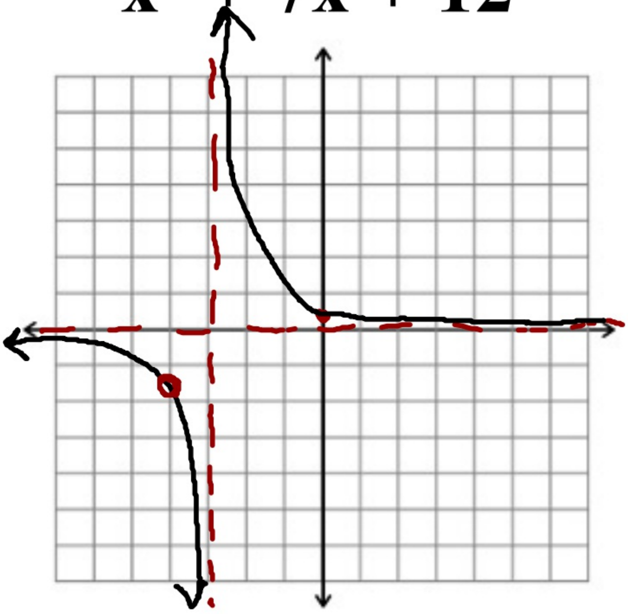
Step 5: Set numerator = 0 to find x-intercepts.

None, because f(x) is undefined at "would-be" x-intercept.

$$f(x) = 0 = \frac{x+4=0}{x^2+7x+12} \rightarrow x = -4 \rightarrow \frac{-4+4}{(-4)^2+7(-4)+12} = \frac{0}{0}, \text{ undefined}$$

Identifying Restrictions & Intercepts

Graph: $f(x) = \dfrac{x + 4}{x^2 + 7x + 12}$



Graphing Guided Practice

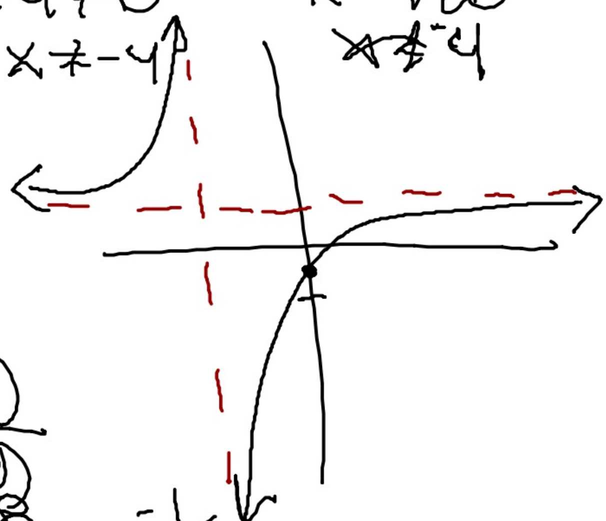
Graph: $f(x) = \dfrac{x^2 + 2x - 8}{x^2 + 8x + 16} = \dfrac{(x-2)(x+4)}{(x+4)(x+4)}$

$x+4 \neq 0$ $\quad x \neq -4$ $\qquad x+4 \neq 0$ $\quad x \neq -4$

Vertical Asymptote(s):

$x \neq -4$

Hole(s):



Horizontal Asymptote(s):

$y = 1$

x-intercept(s): $0 = \dfrac{(x-2)(x+4)}{(x+4)(x+4)}$

$x = 2$

y-intercept(s): $\dfrac{(0-2)(0+4)}{(0+4)(0+4)} = \dfrac{-8}{16} = -\frac{1}{2}$

Independent/Group Work

Worksheet

#s 1-4

****find worksheet online: Assignments, "Graphing Rationals"****

WARM UP

Graph by hand $f(x) = \dfrac{x+5}{x^2-25} = \dfrac{x+5}{(x+5)(x-5)}$

$x+5 \neq 0$

$-5 \quad -5$

$x \neq -5$

1. Vertical Asymptote/Hole:

$x \neq 5$ $\qquad$ $x \neq -5$

b/c it has a matching factor in numerator

2. Horizontal Asymptote:

$y \neq 0$ because the degree in num is less than degree in denom

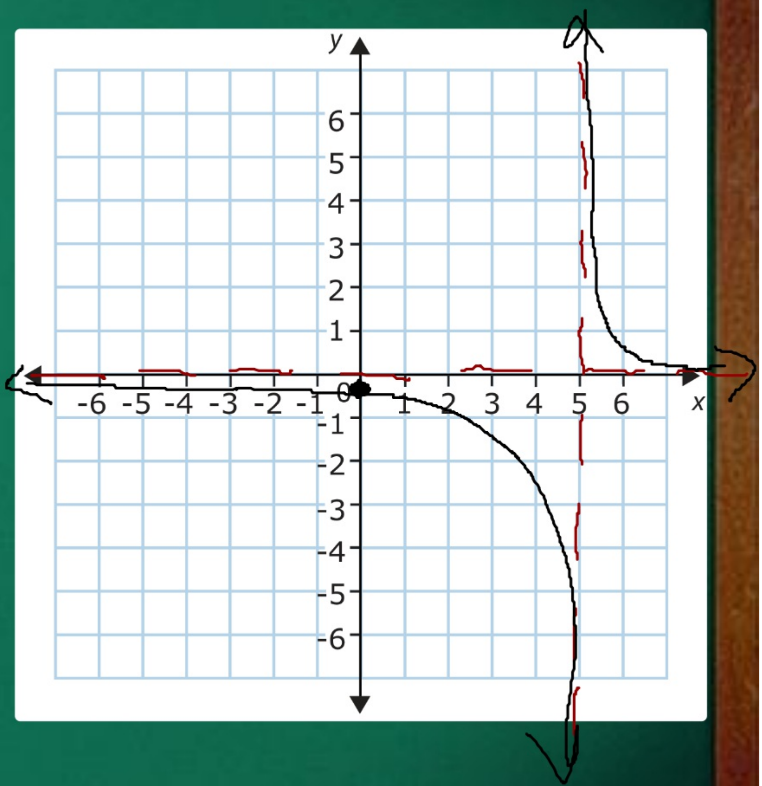
3. Intercepts:

x-int: none because $y \neq 0$ for intercept

y-int: $y = \dfrac{0+5}{(0+5)(0-5)} = \dfrac{5}{-25} = \dfrac{-1}{5}$

$$f(x) = \frac{x+5}{x^2-25}$$

VA: x≠5
Hole: x≠-5
HA: y≠0
x-int: none
y-int: y= -(1/5)

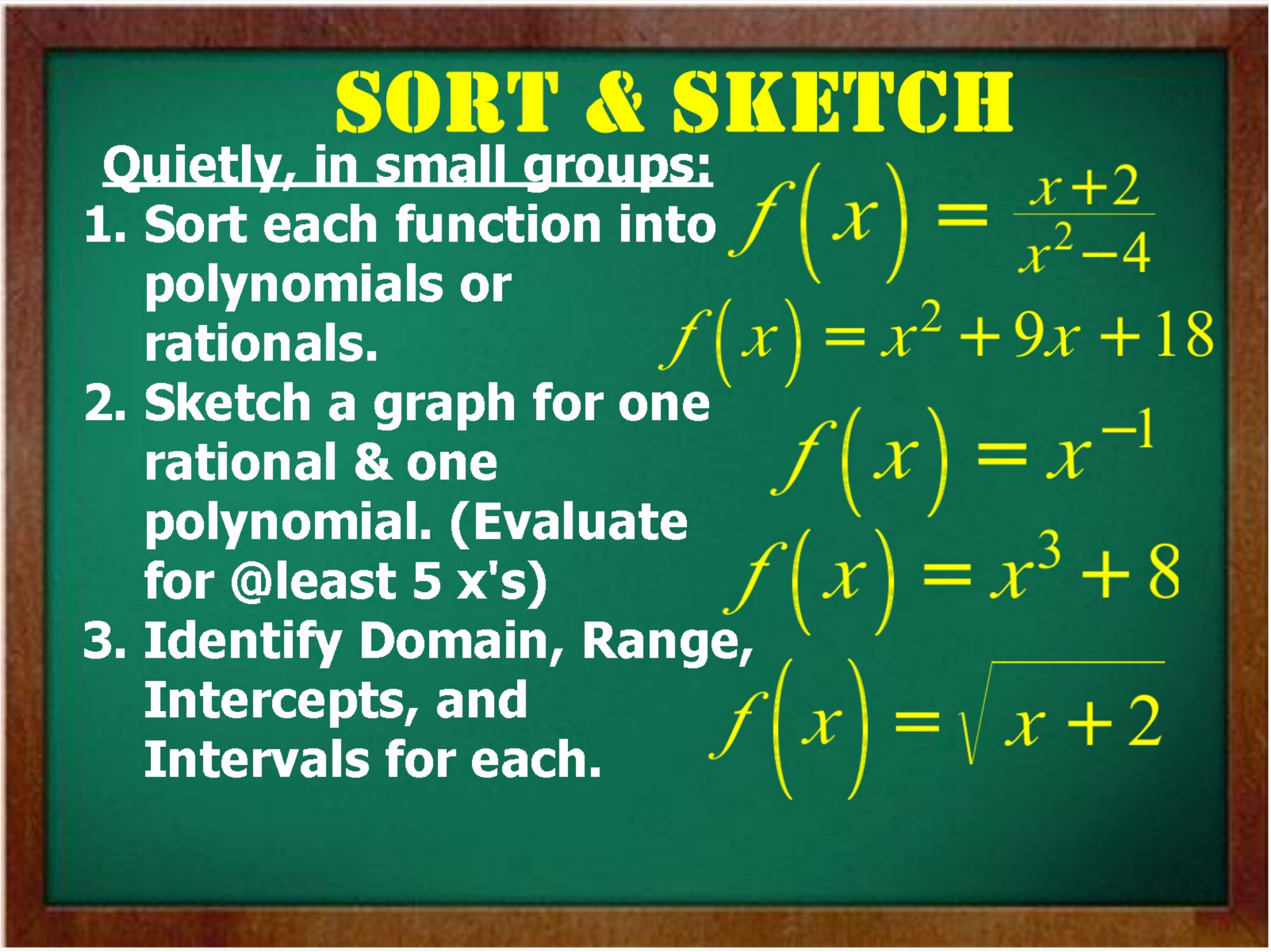


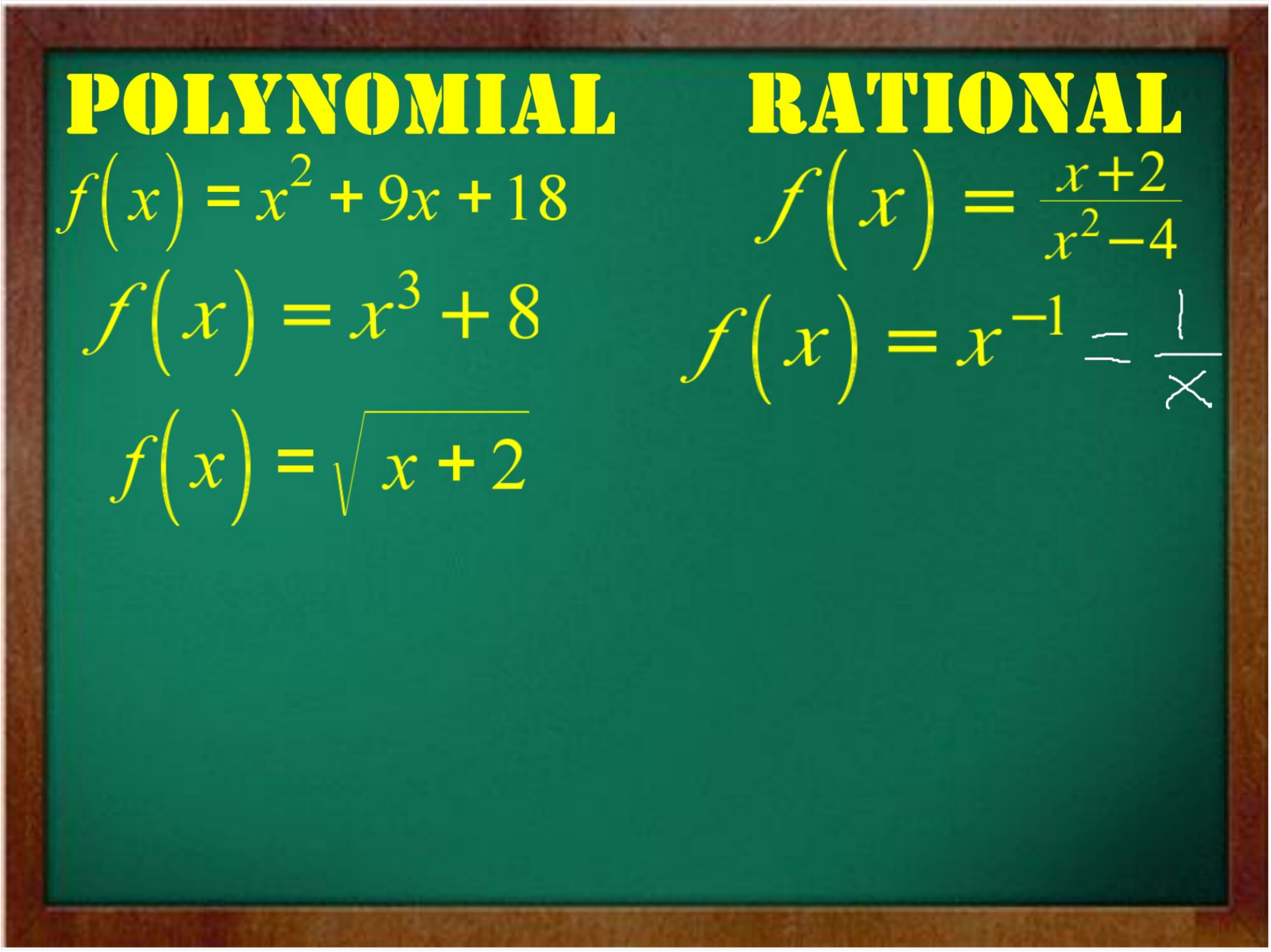
OBJECTIVE

Students will be able to graph piecewise functions and identify key features.

ESSENTIAL QUESTION

How do piecewise functions differ from polynomial and rational graphs?

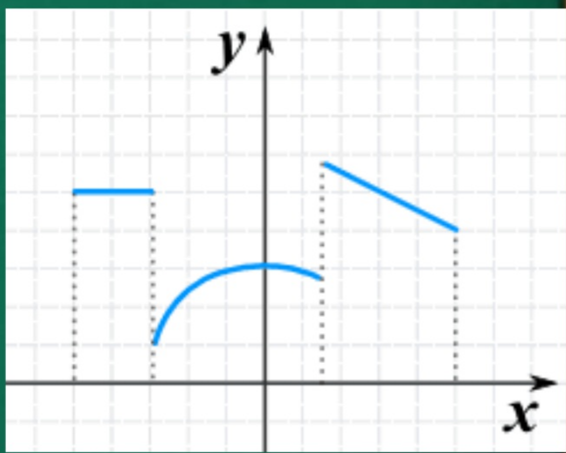

SORT & SKETCH
Quietly, in small groups:
1. Sort each function into polynomials or rationals.
2. Sketch a graph for one rational & one polynomial. (Evaluate for @least 5 x's)
3. Identify Domain, Range, Intercepts, and Intervals for each.
$f(x) = \frac{x+2}{x^2-4}$
$f(x) = x^2 + 9x + 18$
$f(x) = x^{-1}$
$f(x) = x^3 + 8$
$f(x) = \sqrt{x+2}$


POLYNOMIAL
$f(x) = x^2 + 9x + 18$
$f(x) = x^3 + 8$
$f(x) = \sqrt{x + 2}$
RATIONAL
$f(x) = \frac{x+2}{x^2-4}$
$f(x) = x^{-1} = \frac{1}{x}$

PIECEWISE FUNCTIONS

You can create functions that behave differently depending on the input (x) value.

Piecewise Function: acts differently along different pieces of the domain.

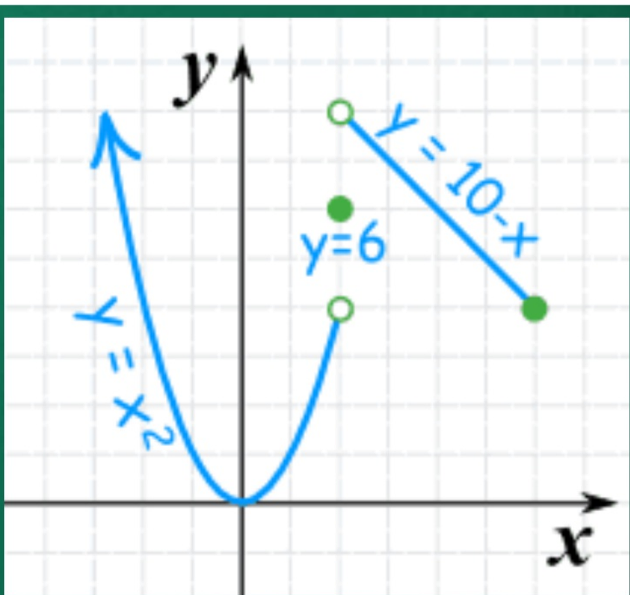


PIECEWISE FUNCTION

Example

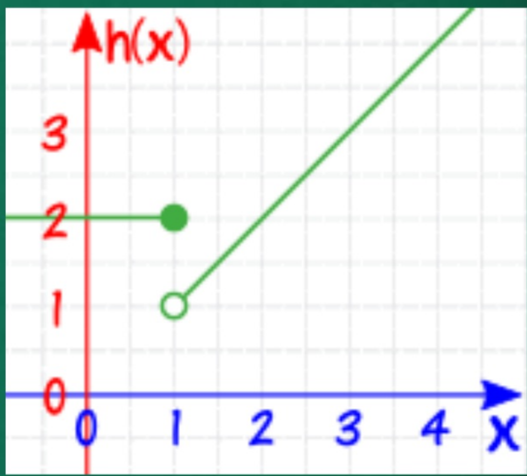
$$f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 6 & \text{if } x = 2 \\ 10 - x & \text{if } x > 2 \text{ and } x \leq 6 \end{cases}$$

X	Y
-4	16
-2	4
0	0
1	1
2	6
3	7



PIECEWISE DISCOVERY

$$h(x) = \begin{cases} 2, & \text{if } x \leq 1 \\ x, & \text{if } x > 1 \end{cases}$$

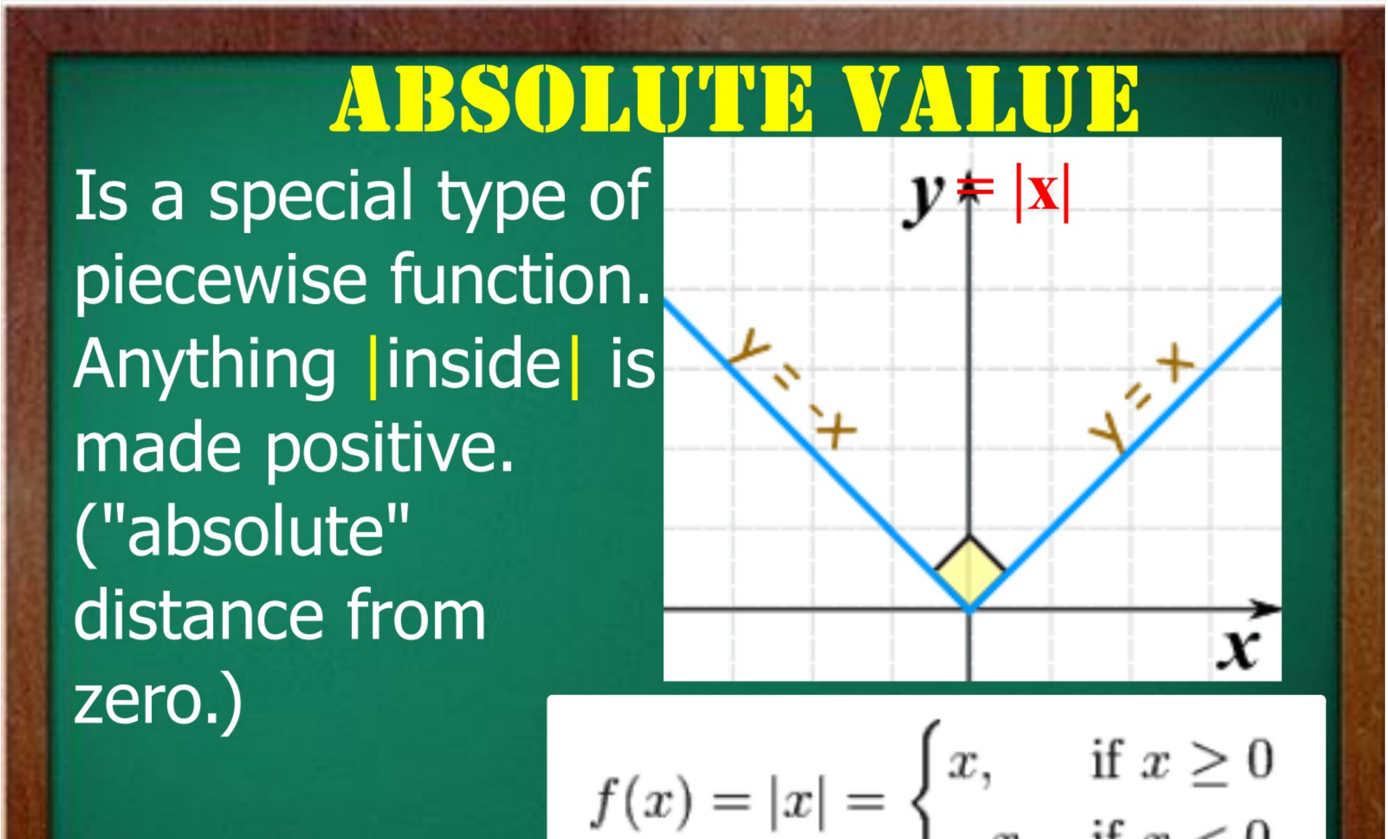


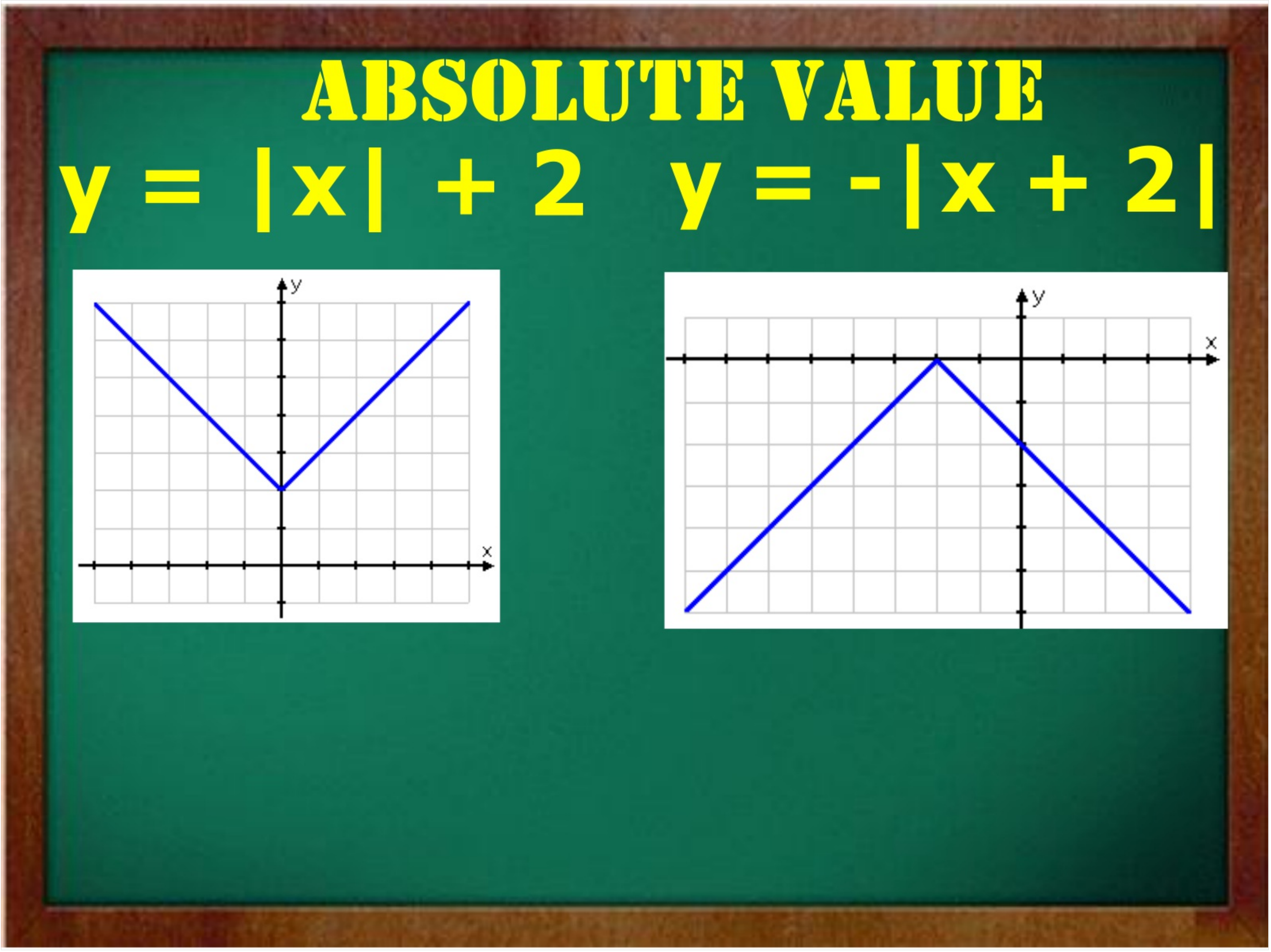
****Find online: Assignments, "Piecewise Discovery"****

UNDERSTAND YOUR BILL

Duke Energy charges .09 cents per kilowatt-hour for the first 200 kWh.The company charges .11 cents per kilowatt-hour for all electrical usage in excess of 200 kWh. How can usage be modeled as a piecewise function.

$$f(x)=\begin{cases} 0.09x, & \text{if } 0 \le x < 200 \\ 0.11x, & \text{if } 200 \le x \end{cases}$$


ABSOLUTE VALUE
Is a special type of piecewise function. Anything |inside| is made positive. ("absolute" distance from zero.)
y = |x|
y = -x
y = x
x
f(x) = |x| = { x, if x ≥ 0; −x, if x < 0

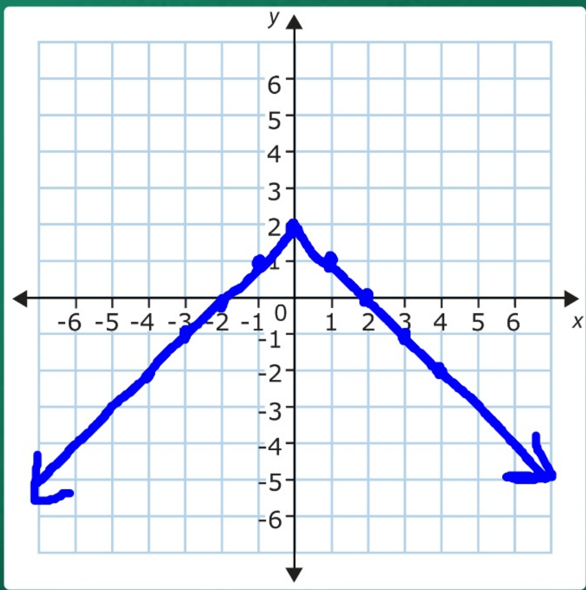


ABSOLUTE VALUE
y = |x| + 2
y = -|x + 2|
y
x
y
x

ABS VAL PRACTICE

Evaluate for at least 5 different poin

sketch graph by hand for $y = -|x| + 2$



x	y
0	2
-1	1
-2	0
1	1
2	0